Application of Integrals

Question 1.

The area bounded by the curves $y = -\sqrt{4 - x^2}$, $x^2 = -\sqrt{2}y$ and x = y is

(a) $\left(\pi + \frac{1}{3}\right)$ sq. units

(b) $\left(\pi - \frac{1}{3}\right)$ sq. units

(c) $\left(\pi + \frac{2}{3}\right)$ sq. units

(d) $\left(\pi - \frac{2}{3}\right)$ sq. units

Answer:

(a) $\left(\pi + \frac{1}{3}\right)$ sq. units

Question 2.

The area common to the ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $0 < b < a$ is

(a) $(a+b)^2 \tan^{-1} \frac{b}{a}$

(b) $(a+b)^2 \tan^{-1} \frac{a}{b}$

(c) $4ab \tan^{-1} \frac{b}{a}$

(d) $4ab \tan^{-1} \frac{a}{b}$

Answer:

(c) $4ab \tan^{-1} \frac{b}{a}$

Question 3.

The area enclosed by the parabola $y^2 = 2x$ and tangents through the point (-2, 0) is

(a) 3 sq. units

(b) 4 sq. units

(c) $\frac{4}{3}$ sq. units

(d) $\frac{8}{3}$ sq. units

Answer:
(d) $\frac{8}{3}$ sq. units

Question 4.
The area bounded by the lines y = 4x + 5, y = 5 − x and 4y = x + 5 is
(a) $\frac{15}{2}$ sq. units
(b) $\frac{9}{2}$ sq. units
(c) $\frac{13}{2}$ sq. units
(d) None of these
Answer:
(a) $\frac{15}{2}$ sq. units

Question 5.
The area bounded by the curves x + 2y^2 = 0 and x + 3y^2 = 1 is
(a) 1 sq. units
(b) $\frac{1}{3}$ sq. units
(c) $\frac{2}{3}$ sq. units
(d) $\frac{4}{3}$ sq. units
Answer:
(d) $\frac{4}{3}$ sq. units

Question 6.
The area bounded by $y = (2x)^{1/2}$ and $x = (2y)^{1/2}$ is
(a) $\frac{4}{3}$ sq. units
(b) $\frac{13}{2}$ sq. units
(c) $\frac{12}{5}$ sq. units
(d) $\frac{4}{25}$ sq. units
Answer:
(a) $\frac{4}{3}$ sq. units

Question 7.
The area of the region {(x, y) : y^2 = x, x^2 + y^2 = 2} is
(a) $\left(\frac{\pi}{4} - \frac{1}{3}\right)$ sq. units
(b) $\left(\frac{\pi}{4} + \frac{1}{3}\right)$ sq. units
(c) $\left(\frac{\pi}{4} - \frac{1}{6}\right)$ sq. units

(d) $\left(\frac{\pi}{2} + \frac{1}{3}\right)$ sq. units

Answer:

(d) $\left(\frac{\pi}{2} + \frac{1}{3}\right)$ sq. units

Question 8.

The area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$ is

(a) $\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ sq. units

(b) $\frac{\sqrt{2}}{6} - \frac{1}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ sq. units

(c) $\frac{3}{2}$ sq. units

(d) $\frac{7}{2}$ sq. units

Answer:

(a) $\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ sq. units

Question 9.

The area bounded by the curve $x^2 = 4y = 4y + 4$ and line $3x + 4y = 0$ is

(a) $\frac{25}{4}$ sq. units

(b) $\frac{125}{8}$ sq. units

(c) $\frac{125}{16}$ sq. units

(d) $\frac{125}{24}$ sq. units

Answer:

(d) $\frac{125}{24}$ sq. units

Question 10.

The area enclosed between the graph of $y = x^3$ and the lines $x = 0$, $y = 1$, $y = 8$ is

(a) $\frac{45}{4}$

(b) 14

(c) 7

(d) none of these

Answer:

(a) $\frac{45}{4}$

Question 11.

The area enclosed by the curve $y = \sqrt{x}$ and $x = -\sqrt{y}$, the circle $x^2 + y^2 = 2$ above the x-axis is

(a) $\frac{\pi}{4}$ sq. units

(b) $\frac{3\pi}{2}$ sq. units

(c) π sq. units

(d) $\frac{\pi}{2}$ sq. units

Answer:

(d) $\frac{\pi}{2}$ sq. units

Question 12.

The ratio in which the x-axis divides the area of the region bounded by the curves $y = x^2 - 4x$ and $y = 2x - x^2$

(a) 4 : 23

(b) 4 : 27

(c) 4 : 19

(d) none of these

Answer:

(a) 4 : 23

Question 13.

Area bounded by the lines $y = |x|$ and $y = 1 - |x - 1|$ is equal to

(a) 4 sq. units

(b) 6 sq. units

(c) 2 sq. units

(d) 8 sq. units

Answer:

(a) 4 sq. units

Question 14.

The area bounded by the lines $y = |x - 1|$ and $y = 3 - |x|$ is

(a) 2 sq. units

(b) 3 sq. units

(c) 4 sq. units

(d) 6 sq. units

Answer:

(c) 4 sq. units

Question 15.

The area bounded by the line $y = 2x - 2$, $y = -x$ and x-axis is given by

(a) $\frac{9}{2}$ sq. units

(b) $\frac{43}{6}$ sq. units

(c) $\frac{35}{6}$ sq. units

(d) None of these

Answer:

(d) None of these

Question 16.

The area of smaller portion bounded by $|y| = -x + 1$ and $y^2 = 4x$ is

(a) 1 sq. units

(b) 2 sq. units

(c) 3 sq. units

(d) none of these

Answer:

(d) none of these

Question 17.

The area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of parabola $y^2 = 4x$ is

(a) $\frac{1}{3}$ $(2 + 3\pi)$ sq. units

(b) $\frac{2}{3}$ $(4 + 3\pi)$ sq. units

(c) $(6 + 3\pi)$ sq. units

(d) $\frac{4}{3}$ $(8 + 3\pi)$ sq. units

Answer:

(d) $\frac{4}{3}$ $(8 + 3\pi)$ sq. units

Question 18.

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

(a) 27 sq. units

(b) 28 sq. units

(c) 54 sq. units

(d) 30 sq. units

Answer:

(a) 27 sq. units

Question 19.

The area included between the curves $x^2 = 4by$ and $y^2 = 4ax$

(a) 16ab sq. units

(b) $\frac{16ab}{3}$ sq. units

(c) 4ab sq. units

(d) $16\pi ab$ sq. units

Answer:

(b) $\frac{16ab}{3}$ sq. units

Question 20.

Area of the region between the curves $x^2 + y^2 = \pi^2$, $y = \sin x$ and y-axis in first quadrant is

(a) $\left(\frac{\pi^3-8}{4}\right)$ sq. units

(b) $\left(\frac{\pi^3-4}{8}\right)$ sq. units

(c) $\left(\frac{\pi^2-8}{4}\right)$ sq. units

(d) $\left(\frac{\pi^2-4}{8}\right)$ sq. units

Answer:

(a) $\left(\frac{\pi^3-8}{4}\right)$ sq. units

Question 21.
If $y = 2 \sin x + \sin 2x$ for $0 \le x \le 2\pi$, then the area enclosed by the curve and x-axis is
(a) $\frac{9}{2}$ sq. units
(b) 8 sq. units
(c) 12 sq. units
(d) 4 sq. units
Answer:
(c) 12 sq. units

Question 22.
The area bounded by the curve $y = x^2 + 4x + 5$, the axes of coordinates and minimum ordinate is
(a) $3\frac{2}{3}$ sq. units
(b) $4\frac{2}{3}$ sq. units
(c) $5\frac{2}{3}$ sq. units
(d) None of these
Answer:
(b) $4\frac{2}{3}$ sq. units

Question 23.
The area of the ellipse $\frac{x^2}{4^2} + \frac{y^2}{9^2} = 1$ is
(a) 6π sq. units
(b) $\frac{\pi(a^2+b^2)}{4}$ sq. units
(c) $p(a + b)$ sq. units
(d) none of these
Answer:
(d) none of these

Question 24.
The area bounded by the curve $2x^2 + y^2 = 2$ is

(a) π sq. units
(b) $\sqrt{2}\pi$ sq. units
(c) $\frac{\pi}{2}$ sq. units
(d) 2π sq. units
Answer:
(b) $\sqrt{2}\pi$ sq. units

Question 25.
Area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
(a) $4\pi ab$ sq.units
(b) $2\pi ab$ sq.units
(c) πab sq.units
(d) $\frac{\pi ab}{2}$ sq.units
Answer:
(c) πab sq.units

Question 26.
Determine the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines x = 0 and x = a.
(a) $\frac{\pi a^a}{4}$
(b) $\frac{\pi a^3}{4}$
(c) $\frac{\pi a^2}{8}$
(d) None of these
Answer:
(a) $\frac{\pi a^a}{4}$

Question 27.
The area enclosed by curve $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is
(a) 10π sq. units
(b) 20π sq. units
(c) 5π sq. units
(d) 4π sq. units
Answer:
(b) 20π sq. units

Question 28.
The area bounded by the curve y = x^2 − 1 and the straight line x + y = 3 is
(a) $\frac{9}{2}$ sq. units
(b) 4 sq. units
(c) $\frac{7\sqrt{17}}{2}$ sq. units

(d) $\frac{17\sqrt{17}}{6}$ sq. units

Answer:

(d) $\frac{17\sqrt{17}}{6}$ sq. units

Question 29.

The area of the region R = ((x, y) : $|x| \leq |y|$ and $x^2 + y^2 \leq 1$) is

(a) $\frac{3\pi}{8}$ sq. units

(b) $\frac{5\pi}{8}$ sq. units

(c) $\frac{\pi}{2}$ sq. units

(d) $\frac{\pi}{8}$ sq. units

Answer:

(c) $\frac{\pi}{2}$ sq. units

Question 30.

The area enclosed between the curve $y^2 = 4x$ and the line y = x is

(a) $\frac{8}{3}$ sq. units

(b) $\frac{4}{3}$ sq. units

(c) $\frac{2}{3}$ sq. units

(d) $\frac{1}{2}$ sq. units

Answer:

(a) $\frac{8}{3}$ sq. units

Question 31.

The area bounded by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$ is

(a) 0 sq. units

(b) $\left(\frac{2\sqrt{2}}{3} + \frac{9\pi}{2} - 9\sin^{-1} \frac{1}{3} \right)$ sq. units

(c) 16π sq. units

(d) None of these

Answer:

(b) $\left(\frac{2\sqrt{2}}{3} + \frac{9\pi}{2} - 9\sin^{-1} \frac{1}{3} \right)$ sq. units

Question 32.

The area bounded by the curves y = sin x, y = cos x and x = 0 is

(a) ($\sqrt{2} - 1$) sq. units

(b) 1 sq. units

(c) $\sqrt{2}$ sq. units

(d) ($1 + \sqrt{2}$) sq. units

Answer:
(a) $(\sqrt{2} - 1)$ sq. units

Question 33.

The area common to the circle $x^2 + y^2 = 16a^2$ and the parabola $y^2 = 6ax$ is

(a) $\frac{4a^2}{3}(4\pi - \sqrt{3})$ sq. units

(b) $\frac{4a^2}{3}(8\pi - 3)$ sq. units   sq. units

(c) $\frac{4a^2}{3}(4\pi + \sqrt{3})$ sq. units

(d) None of these

Answer:
(c) $\frac{4a^2}{3}(4\pi + \sqrt{3})$ sq. units

Question 34.

The area included between curves $y = x^2 - 3x + 2$ and $y = -x^2 + 3x - 2$ is

(a) $\frac{1}{6}$ sq. units

(b) $\frac{1}{2}$ sq. units

(c) 1 sq. units

(d) $\frac{1}{3}$ sq. units

Answer:
(d) $\frac{1}{3}$ sq. units

Question 35.

The area bounded by $x = -4y^2$ and $x - 1 = -5y^2$ is

(a) 1 sq. unit

(b) $\frac{2}{3}$ sq. units

(c) $\frac{2}{3}$ sq. units

(d) 2 sq. units

Answer:
(c) $\frac{2}{3}$ sq. units

Question 36.

The area bounded by the lines $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is

(a) 1 sq. units

(b) 2 sq. units

(c) 3 sq. units

(d) 4 sq. units

Answer:
(b) 2 sq. units

Question 37.

Area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is

(a) $\frac{11}{3}$ sq. units

(b) $\frac{32}{3}$ sq. units

(c) $\frac{43}{3}$ sq. units

(d) $\frac{47}{3}$ sq. units

Answer:

(b) $\frac{32}{3}$ sq. units

Question 38.

Area of the smaller region bounded by $x^2 + y^2 = 9$ and the line $x = 1$ is

(a) $(2 - 3 \sec^{-1} 3)$ sq. units

(b) $(\sqrt{8} - 3\sec^{-1} 3)$ sq.units

(c) $(9\sec^{-1} 3 - \sqrt{8})$ sq. units

(d) $(\sec^{-1} 3 - 3\sqrt{8})$ sq.units

Answer:

(c) $(9\sec^{-1} 3 - \sqrt{8})$ sq. units

Question 39.

The area bounded by the curve $y^2 = x$, line $y = 4$ and y-axis is

(a) $\frac{16}{3}$ sq. units

(b) $\frac{64}{3}$ sq. units

(c) $7\sqrt{2}$ sq. units

(d) none of these

Answer:

(b) $\frac{64}{3}$ sq. units

Question 40.

The area bounded by the curve $x = 3y^2 - 9$ and the line $x = 0$, $y = 0$ and $y = 1$ is

(a) 8 sq. units

(b) $\frac{8}{3}$ sq. units

(c) $\frac{3}{8}$ sq. units

(d) 3 sq. units

Answer:

(a) 8 sq. units

Question 41.

Area bounded by the curve $y^2 = 16x$ and line $y = mx$ is $\frac{2}{3}$ then m is equal to

(a) 3
(b) 4
(c) 1
(d) 2
Answer:
(b) 4

Question 42.
Find the area enclosed by parabola $y^2 = x$ and the line $y + x = 2$ and the x-axis.
(a) $\frac{5}{6}$ sq. units
(b) $\frac{7}{6}$ sq. units
(c) $\frac{6}{7}$ sq. units
(d) $\frac{4}{7}$ sq. units
Answer:
(b) $\frac{7}{6}$ sq. units

Question 43.
The area bounded by the curve $x^2 + y^2 = 1$ and 1st quadrant is
(a) $\frac{\pi}{4}$ sq.units
(b) $\frac{\pi}{2}$ sq. units
(c) $\frac{\pi}{3}$ sq.units
(d) $\frac{\pi}{6}$ sq.units
Answer:
(a) $\frac{\pi}{4}$ sq.units

Question 44.
Area bounded by the curve $y = \cos x$ between $x = 0$ and $x = \frac{3\pi}{2}$ is
(a) 1 sq. units
(b) 2 sq. units
(c) 3 sq. units
(d) 4 sq. units
Answer:
(c) 3 sq. units

Question 45.
The area of the region bounded by the curve $y = \sqrt{4 - x^2}$ and x-axis is
(a) 8π sq. units
(b) 2π sq. units
(c) 16π sq. units
(d) 6π sq. units

Answer:
(b) 2π sq. units